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# Technical Notes

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## Wall Curvature Effects on External Burning in Supersonic Flow

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### Nomenclature

| | |
|---|---|
| $A, B, b, c$ | = coefficients, see Eqs. (9–11) |
| $H$ | = nondimensional $(\Delta T^0)_c$, $(\Delta T^0)_c / T^{0*}$ |
| $h$ | = normal section height |
| $K$ | = coefficient, see Eq. (3) |
| $M$ | = Mach number |
| $\dot{m}$ | = mass flow |
| $P$ | = static pressure |
| $s$ | = wall abscissa |
| $T^0$ | = total temperature |
| $U$ | = velocity |
| $x$ | = nondimensional wall abscissa, $s/h^*$ |
| $w$ | = nondimensional mass flow, $\dot{m}/\dot{m}^*$ |
| $\alpha$ | = nondimensional normal section height, $h/h^*$ |
| $\beta$ | = nondimensional pressure, $P/P_1$ |
| $\gamma$ | = specific heat ratio |
| $(\Delta T^0)_c$ | = stoichiometric inner/outer flow mixture adiabatic combustion total temperature increase |
| $\theta$ | = local external stream deflection angle |
| $\theta_w$ | = local wall slope angle |
| $\nu$ | = velocity ratio, $U_1/U$ |
| $\xi$ | = nondimensional wall abscissa, $s/h_i$ |
| $\tau$ | = nondimensional total temperature, $T^0/T^{0*}$ |
| $\varphi$ | = momentum ratio, $(\rho U)/(\rho_1 U_1)$ |

*Subscripts*

| | |
|---|---|
| $i$ | = inflow station |
| 1 | = undisturbed outer stream |

*Superscript*

| | |
|---|---|
| $*$ | = viscous-throat station |

### Introduction

THE present work is based on the very ingenious models that the Billig school has produced and developed over the last two decades concerning combustion in the viscous mixing region along surfaces exposed to a supersonic external flow. More precisely, reference is made to the analysis presented in 1994 (Ref. 1) that is mainly devoted to delineating the limitations of the allowable heating of the boundary layer, which are posed by the possibility of a smooth subsonic to supersonic, or vice versa, transition at the viscous throat. The authors of this Note have reexamined that analysis and have tried to highlight the limitations of the heating amounts that arise from the surface geometry. As for applications, we have also envisaged a way of recognizing the throat position on a predetermined curved surface and of obtaining the complete solution that matches the initial conditions.

The thermo–aerodynamic problem, schematically shown in Fig. 1, is that of a fuel rich inner stream flowing along a contoured wall and exposed on the other side to a supersonic stream of pure or vitiated air. Air entrainment from the outer stream permits combustion to take place inside the inner stream, thus also influencing the inner stream development.

The ingenious model proposed[1] for the process, and here retained, is not detailed again. Just keep in mind that the inner stream flow is treated as a one-dimensional flow, with instantaneous full mixing of the freestream fluid entrained into the inner stream and with instantaneous combustion of the fuel in the stoichiometric ratio with the entrained air. The outer supersonic flow evolves as an inviscid flow, matching the pressure with the inner flow pressure, station by station. Constant calorific properties of the fluid are assumed. Heating from and viscous stresses on the wall are neglected.

### Equations

The equations that model the process in the planar flow case are obtained from Ref. 1, and are written here, with some refinements, directly in nondimensional terms. The quantities $h$, $\dot{m}$, and $T^0$ have all been made nondimensional by using the values that they achieve at the viscous–thermal throat station, whereas the pressure is referenced to the static undisturbed pressure of the external flow $P_1$, that is, the external flow pressure before the two stream confluence. $T_1^0$ is referenced to $T^{0*}$. Angles $\theta_w$ and $\theta$ are measured from the undisturbed external stream direction and are taken as positive in the outward direction. Note that the derivatives are considered in the variable $x$, that is, the wall abscissa $s$, measured from the inflow station, where the two streams come into contact, and are nondimensionalized using $h^*$. Because, in general, the actual wall geometry, in particular, the wall slope $\theta_w$, is assigned as a function of $\xi$ (that is, of

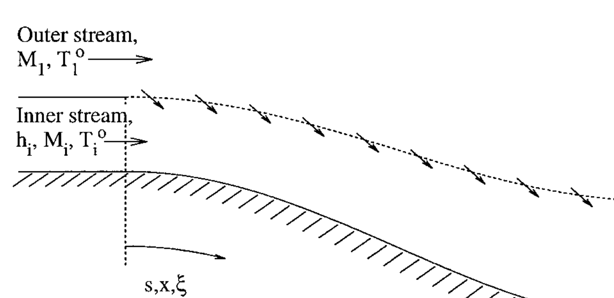


**Fig. 1 Schematic of the flowfield.**

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the wall abscissa $s$, nondimensionalized by the a priori known height $h_i$) some problems arise when integrating the equations: This will again be referred to when treating the applications. The equations are

$$\frac{\mathrm{d}M^2}{\mathrm{d}x} = \frac{M^2[2+(\gamma-1)M^2]}{M^2-1}\left\{\frac{1}{\alpha}\frac{\mathrm{d}\alpha}{\mathrm{d}x} - \frac{1+\gamma M^2}{2}\frac{1}{\tau}\frac{\mathrm{d}\tau}{\mathrm{d}x} + [\gamma M^2(\nu-1)-1]\frac{1}{w}\frac{\mathrm{d}w}{\mathrm{d}x}\right\} \tag{1}$$

$$\frac{\mathrm{d}\tau}{\mathrm{d}x} = (\tau_1 + H - \tau)\frac{1}{w}\frac{\mathrm{d}w}{\mathrm{d}x} \tag{2}$$

$$\frac{\mathrm{d}w}{\mathrm{d}x} = K\frac{1-\varphi}{\varphi}\frac{w}{\alpha} \tag{3}$$

$$\frac{\mathrm{d}\alpha}{\mathrm{d}x} = \theta + K(1-\varphi) - \theta_w \tag{4}$$

Equation (1) describes the inner flow Mach number variation on the basis of the conservation equations. Equation (2) takes into account the total temperature variation due to the combined effects of the combustion heat release and of the freestream entrainment into the mixing zone. The parameter $H$ is the nondimensional form (referenced to $T^{0*}$) of the total temperature increase $(\Delta T^0)_c$ that a stoichiometric mixture of the (possibly vitiated) external air and the inner flow fuel would experience in an adiabatic combustion. Equation (3) describes the turbulent air mass entrainment that is the crucial element of the overall model; a constant value of 0.02 is suggested[1] for the $K$ coefficient. Equation (4) represents the skillful model[1] of the relation between the pressure and mass entrainment. To determine $\nu$, $\varphi$, and $\beta$ write

$$\nu = \sqrt{\frac{\tau_1}{\tau}\frac{M_1^2}{M^2}\frac{2+(\gamma-1)M^2}{2+(\gamma-1)M_1^2}}, \qquad \varphi = \frac{M^2}{M_1^2}\nu\beta$$
$$\beta = \beta^*\frac{w}{\alpha}\sqrt{\frac{\tau}{M^2}\frac{\gamma+1}{2+(\gamma-1)M^2}} \tag{5}$$

whereas $\theta$ can be derived by applying the linearized supersonic theory

$$\theta = \frac{\sqrt{M_1^2-1}}{\gamma M_1^2}\ln\beta \tag{6}$$

## Viscous-Throat Conditions

The mathematical problem is posed by giving values to the $\gamma$, $M_1$, $\tau_1$, and $H$ parameters and by assigning the wall geometry (note that all of the following calculations were made assuming $\gamma = 1.4$, to be in accordance with Ref. 1). Equation (1) becomes singular when a sonic condition arises. Therefore, the integration process must be split into a backward and a forward leg starting from the throat station, where the initial conditions, that is, $\beta^*$ and $(\mathrm{d}M^2/\mathrm{d}x)^*$, must be established beforehand. To have a regular transition through Mach 1, the vanishing of the term in braces in Eq. (1) is required. This condition is the key to setting the throat pressure $\beta^*$ once the wall slope $\theta_w^*$ at the throat is assumed. Namely, for $\beta^*$ one obtains

$$\beta^{*2} - \frac{M_1^2}{\nu^*}\left(A + 1 - \frac{\theta_w^*}{K}\right)\beta^* + A\left(\frac{M_1^2}{\nu^*}\right)^2 = \frac{M_1^2}{K\nu^*}\frac{\sqrt{M_1^2-1}}{\gamma M_1^2}\beta^*\ln\beta^* \tag{7}$$

Then, by applying the L'Hospital rule to Eq. (1), one determines the throat value of the Mach number derivative, thus obtaining the means to start the integration process. The thus obtained equation is a second-degree algebraic equation on $(\mathrm{d}M^2/\mathrm{d}x)^*$, just as in all choking situations when treating one-dimensional flows. Write

$$\left[\left(\frac{\mathrm{d}M^2}{\mathrm{d}x}\right)^*\right]^2 + b\left(\frac{\mathrm{d}M^2}{\mathrm{d}x}\right)^* + c = 0 \tag{8}$$

The $b$ and $c$ coefficients take on the expressions

$$b = \frac{\sqrt{M_1^2-1}}{M_1^2} + \gamma\frac{\gamma+1}{2}\left(\frac{\mathrm{d}\tau}{\mathrm{d}x}\right)^* + \gamma(\gamma+1-\gamma\nu^*)\left(\frac{\mathrm{d}w}{\mathrm{d}x}\right)^* \tag{9}$$

$$c = \left\{K\left(\frac{\mathrm{d}\varphi}{\mathrm{d}x}\right)^* + \left(\frac{\mathrm{d}\alpha}{\mathrm{d}x}\right)^{*2} + \left(\frac{\mathrm{d}\theta_w}{\mathrm{d}x}\right)^* - \frac{\sqrt{M_1^2-1}}{\gamma M_1^2}\left[\frac{1}{2}\left(\frac{\mathrm{d}\tau}{\mathrm{d}x}\right)^* + \left(\frac{1}{\varphi}\frac{\mathrm{d}\varphi}{\mathrm{d}x}\right)^*\right] + A\left[\left(\frac{\mathrm{d}^2w}{\mathrm{d}x^2}\right)^* - \left(\frac{\mathrm{d}w}{\mathrm{d}x}\right)^{*2}\right] + B\left(\frac{\mathrm{d}\tau}{\mathrm{d}x}\right)^*\left(\frac{\mathrm{d}w}{\mathrm{d}x}\right)^*\right\}(\gamma+1) \tag{10}$$

The aforementioned coefficients $A$ and $B$ are

$$A = [(\gamma+1)/2](\tau_1 + H + 1) - \gamma\nu^*$$
$$B = (\gamma/2)\nu^* - [(\gamma+1)/2](\tau_1 + H) \tag{11}$$

The $b$ coefficient results to be positive (at least in a large range of the parameter values and when dealing with heating problems). Therefore, one is assured that, if real solutions exist, at least one is negative, and this means transition from supersonic to subsonic flow through the throat is possible. The sign of the $c$ coefficient is then conclusive as far as the possibility of the inverse transition from subsonic to supersonic flow is concerned. If it is negative, this possibility is obtained; otherwise only passages of the supersonic–subsonic type are admitted. The vanishing of $c$ discriminates these two cases. Therefore, the $c = 0$ curve in the heating parameter plane $[T^{0*}/T_1^0, (\Delta T^0)_c/T_1^0]$ separates the region where one can have a subsonic to supersonic transition (the region below the curve) from that where only supersonic to subsonic transition is possible. The $(\mathrm{d}\theta_w/\mathrm{d}x)^*$ term in Eq. (10) represents the influence of the wall curvature at the viscous throat on the $c$ coefficient. It has a considerable effect on the extension of the aforementioned region. Schetz et al.[1] have complained about the great limitations of allowable heat release, implied by throat conditions. They have only considered cases where the curvature at the viscous-throat position is zero. However, the wall curvature is quite effective in relaxing the heat release limitation. This can be seen by comparing the region below the $c = 0$ curve (Fig. 2) for a case where the wall curvature at the viscous throat is zero with that when the wall curvature assumes a negative value [in Fig. 2, $(\mathrm{d}\theta_w/\mathrm{d}x)^* = -0.01$]. The $(\Delta T^0)_c$ limit results are more than doubled when $\theta_w^* = 0$. A lower (but yet remarkable) increment appears when $\theta_w^* < 0$ because, as Fig. 2 shows, although negative

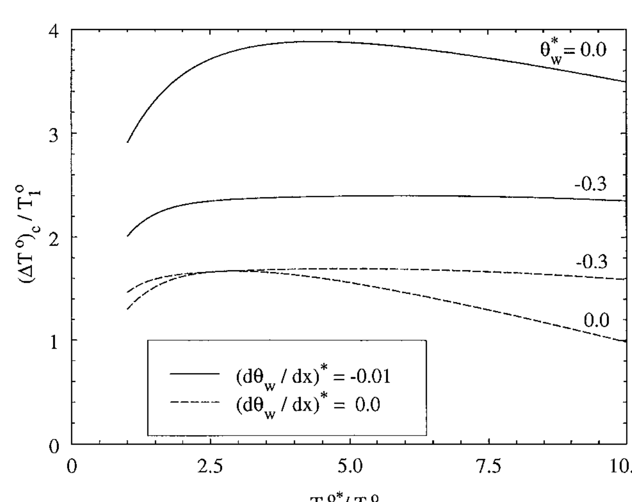


**Fig. 2 Limitations on allowable heat release implied by conditions at the viscous throat ; $M_1 = 2$.**

wall angles move the $c = 0$ boundary up if the wall curvature is zero, they move it down if a negative curvature is assumed.

## Applications

In any particular problem that must be faced in applications, precise values of some parameters are assigned at the inflow station. Besides the Mach number $M_1$, total temperatures $T_1^0$ and $T_i^0$ are given (it is in fact sufficient to give their ratio $T_i^0/T_1^0 = \tau_i/\tau_1$). A relationship between the pressure $\beta_i$ and the Mach number $M_i$ of the inner flow must also be assigned, or (as in the examples given hereafter) just pressure $\beta_i$ may be fixed. Moreover, the heating capability of the fuel is assessed by giving a value of the $(\Delta T^0)_c$ parameter [it is sufficient to give the ratio $(\Delta T^0)_c/T_1^0 = H/\tau_1$].

The wall geometry must be given next. Clearly, $\theta_w$ cannot be given as a function of the $x = s/h^*$ integration variable because $h^*$ is not known a priori. On the contrary, it is generally given as a function of the $\xi = s/h_i$ variable. Therefore, when integrating the equations backward from the throat, one has to anticipate the value of $\alpha_i = h_i/h^*$ that is achieved at the inflow station, to be able to read the geometry in the $x$ variable language. This is obviously not the case when $\theta_w$ is a constant, that is, when dealing with straight walls, but constitutes a peculiar aspect of applications with curved walls.

The integration of the system of Eqs. (1), (3), and (4) backward from the throat would, therefore, appear to be a two-point boundary-value problem. One has to reach specified values of the $\beta$ and $\tau$ parameters at the inflow station, at the same time the $\alpha$ variable takes on an earlier assumed $\alpha_i$ value there. Consequently the throat position $\xi^*$ and $H$ heating parameter (hence, $\tau_1$) are obtained. Numerical difficulties can arise due to the system stiffness. As pointed out in Ref. 1 and as also experienced by the present authors, the equation system is often stiff and requires ad hoc integration procedures, based on implicit methods (such integration routines may be found in literature[2]). Usually, but not always, the integration proceedes safely this way, provided that the throat station has been left back some distance, thus achieving a Mach number value that is a few points different from 1. The Euler method (a first-order integration step is sufficient) is used in this first phase.

To highlight the problem peculiarities and the solution method, the authors have taken as an example the case of a curved wall where

$$\theta_w = -(\pi/10)\sin[(\pi/20)\xi] \tag{12}$$

and have assumed that the outer flow does not exhibit either an expansion pressure fan or an oblique shock at the inflow station, so that the inner flow pressure is at the undisturbed outer flow pressure level, $\beta_i = 1$. Only the transition from subsonic to supersonic flow has been envisaged. A code[3] for treating optimal control problems using indirect methods (where boundary-value problems are quite familiar) and capable of facing stiffness problems was profitably used.

Solutions have been found for a set of $T_i^0/T_1^0$ and a range of $(\Delta T^0)_c/T_1^0$ problem defining parameters, obtaining the relative values of the viscous throat position $\xi^*$ of the heating of the inner flow from the inlet station to the throat (given by $1/\tau_1$), of the throat area given by $\alpha_i$, and of the compatible inlet station Mach number $M_i$. For conciseness, only the $\xi^*$ and $M_i$ values are shown in Fig. 3. It appears that, when increasing the $(\Delta T^0)_c/T_1^0$ with a given $T_i^0/T_1^0$ value, the attendant increasing heating rate pushes the viscous throat away from the inlet station [see Eqs. (1) and (4): to null the term in braces in Eq. (1), $d\alpha/dx$ must reach higher values, which are obtained where $-\theta_w$ is larger]. The allowable value of the inlet station Mach number $M_i$ (related to the nondimensional mass flow parameter) then gradually decreases, somewhat recalling the inlet Mach number limit behavior in the Rayleigh flow.

The complete flow behavior along the wall is then represented (Fig. 4) for a particular choice of $T_i^0/T_1^0$ and $(\Delta T^0)_c/T_1^0$ parameters. Note the smooth transition through the throat of the curves: This, by itself, is a reason to have confidence in the integration process. Not even the integration accuracy is questioned by the irregular behavior that the Mach number curve shows downstream to the throat, where it appears to again pass through Mach 1. This is because, due to wall curvature, the outer stream pressure (hence, the inner one) stops dropping and possibly starts to rise again. This contrasts the Mach number increase and brings it back to the sonic value. The flow, however, no longer meets the regular throat conditions there, and the equations fail to furnish a regular solution. Physically, one expects a shock to be formed and stay somewhere downstream to the throat, again leaving a subsonic flow.

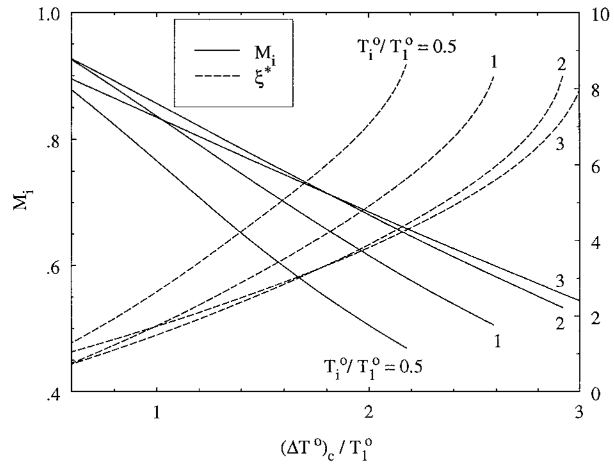


**Fig. 3 Calculated throat position and allowable inflow Mach number for the Eq. (12) wall geometry; $M_1$ = 2.**

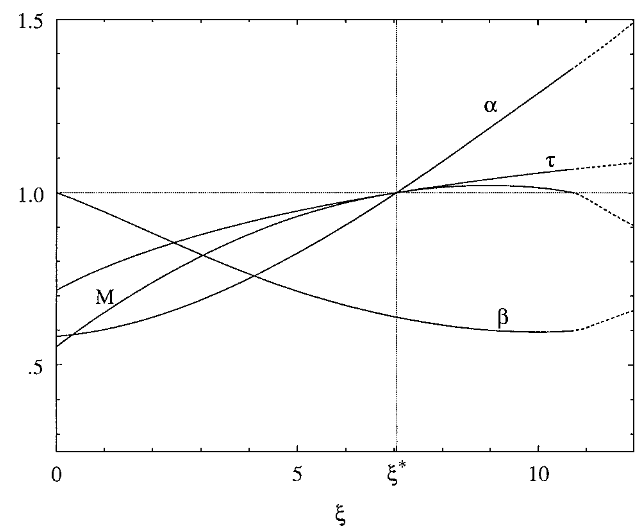


**Fig. 4 Calculated flowfield for $T_i^0/T_1^0$ = 2 and $(\Delta T^0)_c/T_1^0$ = 2.8 on the Eq. (12) curved wall; $M_1$ = 2.**

## Conclusions

In revisiting the analysis[1] of external burning in supersonic flow, two particular areas have been addressed.

First, the strong wall curvature effect on enlargement of the set of possible heating parameters has been shown. External burning on curved surfaces (which are more likely of interest in applications) appears to be a more promising possibility than that on straight walls.

Second, for applications on curved surfaces, the most striking novelty in contrast to the straight wall cases, that is, the necessity of reading the wall slope in terms of the integration variable, has been addressed and solved. The use of boundary-value problem codes to recognize the throat position on the wall and to obtain proper values of the parameters at the inflow station have proven to be very useful and powerful.

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# Three-Dimensional Disturbance Vortex Method for Simulating Rotor/Stator Interaction in Turbomachinery

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## I. Introduction

IT is necessary to account for three-dimensional unsteady flow effects in the design procedure to improve turbomachine performance and efficiency. The present trend toward closer blade-row spacing has emphasized the need for reliable techniques to resolve the unsteady flowfield. There are three dominant causes of unsteadiness relating to rotor and stator interaction. The first is the interaction between wakes and downstream blades. The second is the potential interaction between upstream and downstream blades. The third is the interaction between upstream vortex and downstream blades. For modern transonic turbomachines, as pointed out by Jung et al.,[1] if the velocity is sufficiently high at the interface between rotor and stator, the effects of upstream unsteadiness on the downstream are much stronger than that of the downstream on the upstream, and the disturbance is dominantly transported downstream. The disturbance vortex method for simulating two-dimensional rotor/stator interaction in turbomachinery has been presented by Wu and Chen,[2] where good agreement between the computational and experimental results is obtained. Therefore, it is natural to extend this method to three-dimensional domain. In view of its inherent advantages, such as treatment of the complex boundaries and application of the Baldwin–Lomax[3] turbulence model to wakes, the idea is developed to simulate three-dimensional unsteady flow in turbomachinery.

## II. Numerical Method

### Governing Equations

The vorticity dynamic equation for three-dimensional viscous flow is

$$\frac{\partial \omega}{\partial t} + (V \cdot \nabla)\omega = (\omega \cdot \nabla)V - \omega(\nabla \cdot V) + \nu\nabla^2\omega \qquad (1)$$

Substituting $\omega$ and $V$ in Eq. (1) by $\omega = \bar{\Omega} + \omega'$ and $V = \bar{U} + u'$, respectively, one obtains

$$\frac{\partial \bar{\Omega}}{\partial t} + \frac{\partial \omega'}{\partial t} + (\bar{U} \cdot \nabla)\bar{\Omega} + (\bar{U} \cdot \nabla)\omega' + (u' \cdot \nabla)\bar{\Omega} + (u' \cdot \nabla)\omega'$$
$$= (\bar{\Omega} \cdot \nabla)\bar{U} + (\bar{\Omega} \cdot \nabla)u' + (\omega' \cdot \nabla)\bar{U} + (\omega' \cdot \nabla)u' - \bar{\Omega}(\nabla \cdot \bar{U})$$
$$-\bar{\Omega}(\nabla \cdot u') - \omega'(\nabla \cdot \bar{U}) - \omega'(\nabla \cdot u') + \nu\nabla^2\bar{\Omega} + \nu\nabla^2\omega' \qquad (2)$$

where $\bar{\Omega} = \nabla \times \bar{U}$ is time-averaged vorticity and $\omega' = \nabla \times u'$ is disturbance vorticity. If the Mach number of time-averaged velocity is not very high, we can assume the volume expansion corresponding to disturbance velocities is zero,

$$\nabla \cdot u' = 0 \qquad (3)$$

Further discussion about this assumption may be found in Ref. 2.

Performing a time-averaging operation to Eq. (2) and applying

$$\int_0^T q' \, \mathrm{d}t = 0, \qquad (q = \omega, V)$$

the equation for disturbance vorticity can be obtained as

$$\frac{\mathrm{d}\omega'}{\mathrm{d}t} = -(u' \cdot \nabla)\bar{\Omega} + \overline{(u' \cdot \nabla)\omega'} + (\bar{\Omega} \cdot \nabla)u' + (\omega' \cdot \nabla)\bar{U}$$
$$+ (\omega' \cdot \nabla)u' - \overline{(\omega' \cdot \nabla)u'} - \omega'(\nabla \cdot \bar{U}) + \nu\nabla^2\omega' \qquad (4)$$

where $\mathrm{d}/\mathrm{d}t$ is the material derivative. In the case of laminar flow, the viscosity $\nu = \nu_l$, whereas for turbulent flow $\nu = (\nu_l + \nu_t)$, where $\nu_l$ and $\nu_t$ are the laminar and turbulent viscosity.

### Initial and Boundary Conditions

The initial condition can be specified as follows:

$$q'(r, \theta, z, t_0) = \begin{cases} \bar{Q}_r(r, \theta, z) - \bar{Q}_{(\mathrm{axi})}(r, z) & x = x_1 \\ 0 & x \neq x_1 \end{cases} \qquad (5)$$

where $\bar{Q}_{(\mathrm{axi})}$ is the circumferential averaged value of $\bar{Q}_r$, the relative steady solutions of the upstream rotor; $x(r, \theta, z)$ is the coordinate; and $x_1$ are the inlet points of the stator domain.

On the solid wall, the impenetrable and no-slip conditions should be satisfied (discussed subsequently).

At upstream boundary, the upstream boundary condition for disturbance variables at time $t$ can be obtained in the same way as Eq. (5) is obtained,

$$q'(r, \theta, z, t) = \bar{Q}_r(r, \theta - \lambda t, z) - \bar{Q}_{(\mathrm{axi})}(r, z) \qquad (6)$$

where $t = t_0 + k\Delta t$, $(1 \le k \le M)$, $\Delta t = T/M$, $\lambda$ is the angular velocity of the rotor, and $T = P_r/V_r$, is the time period where $P_r$ is rotor blade spacing, $V_r$ is the circumferential velocity of the rotor, and $M$ is the number of time steps in one period.

For geometric boundaries in a single passage, if the rotor and stator have the same blade spacing, a simple periodic condition exists. If the blade spacings are different, for example, the rotor spacing is larger than that of the stator (see Fig. 1), the "phase-shift periodic boundary condition"[4] exists:

$$q(r, \theta, z, t) = q(r, \theta + \theta_s, z, t + \Delta T) \qquad (7)$$

where $\Delta T = (P_r - P_s)/V_r$, $\theta_s = P_s/r$, and $P_s$ is the stator blade spacing.

### Turbulence Model

As in two-dimensional domain,[2] the Baldwin and Lomax[3] turbulence model faces the same difficulties when it is used to calculate unsteady wake. One difficulty is that the computed viscosity in the outer region is much higher than that in the boundary regions. This is because there is a high vorticity region in the wakes so that $F(y)$,

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